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RESONANT AND NON-RESONANT PIECES OF THE $D \rightarrow \bar{K}\pi$ SEMILEPTONIC TRANSITION AMPLITUDE

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Abstract

We compare the resonant and non-resonant contributions in various regions of phase space for the $D \rightarrow \bar{K}\pi$ semileptonic transition amplitude, computed in a chiral model which incorporates the heavy quark symmetry. Remarks on the significance for experiment and for chiral perturbation theory are made.

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1. INTRODUCTION

In this note we shall examine the weak hadronic current matrix element for the decay $D^0 \rightarrow K^- \pi^0 e^+ \nu_e$ using a chiral Lagrangian which incorporates the heavy quark symmetry. Previous works have treated this process in a chiral model which includes only light pseudoscalars [1] and in chiral models with both light pseudoscalars and light vectors present but with the approximation that the decay be replaced by $D^0 \rightarrow K^{*-} e^+ \nu_e$ [2-6]. Here we will consider *both* contributions together. This is interesting, of course, in its own right. It also holds some interest for the question of what is the best way to incorporate vector mesons in the chiral perturbation theory program [7]. It may actually be easier to investigate this question in the framework of chiral light-heavy interactions rather than light-light interactions since the “heavy end” might eventually be under better control. For the present process we find that there is no region of phase space in which the light vector K^* piece does not make a non-negligible contribution and that it is typically very dominant. This perhaps suggests the adoption of a framework in which light pseudoscalars and light vectors are treated together from the beginning.

The detailed points include the discussion of the way the chiral theorem is maintained in the appropriate unphysical limit and the treatment of the phase space kinematics for the hadronic matrix element. The transition amplitude is given in Section 2 (see also the Appendix) after our notation has been introduced. Section 3 contains the kinematics and the comparison of the resonant and non-resonant pieces of the amplitude in various regions of phase space. Some caveats and remarks on the experimental aspect of K^* dominance are given in Section 4.

2. Decay Amplitudes

We are interested in the chiral invariant interactions of both the light pseudoscalar nonet $\phi(x)$ and the light vector nonet $\rho_\mu(x)$ with the heavy meson field. We shall follow the notation of Ref. [3]; other treatments include Refs. [2, 4, 5, 6]. The chiral interactions involving only the light pseudoscalars were discussed in Ref. [8]. Using the “heavy field” $H(x)$ which contains both heavy pseudoscalar as well as heavy vector pieces, the leading order (in heavy meson mass M and in number of derivatives of the light fields) strong interaction is compactly written as [3]:

$$\begin{aligned} \frac{1}{M} \mathcal{L}_{\text{light} - \text{heavy}} &= iV_\mu \text{Tr} \{H[\partial_\mu - i\alpha\tilde{g}\rho_\mu - i(1-\alpha)v_\mu]\overline{H}\} \\ &+ id \text{Tr} [H\gamma_\mu\gamma_5 p_\mu\overline{H}] + \frac{ic}{m_v} \text{Tr} [H\gamma_\mu\gamma_\nu F_{\mu\nu}(\rho)\overline{H}], \end{aligned} \quad (2.1)$$

wherein m_v is the light vector meson mass introduced just to keep the coupling constant c dimensionless and

$$v_\mu, p_\mu = \frac{i}{2}(\xi\partial_\mu\xi^\dagger \pm \xi^\dagger\partial_\mu\xi), \quad (2.2)$$

with the chiral matrix $\xi = \exp(i\phi/F_\pi)$. Furthermore our normalization convention sets $F_\pi \simeq 132$ MeV. V_μ is the heavy meson 4-velocity and the heavy field H here is taken to have the canonical dimension of one. $\tilde{g} \simeq 3.93$ is the light vector-light pseudoscalars coupling constant. Heavy quark symmetry breaking terms, SU(3) symmetry breaking terms as well as the chiral Lagrangian of the light sector have not been explicitly written.

Notice that the light-heavy interaction (2.1) is characterized by the three dimensionless coupling constants α , c and d (denoted g in [8]). The choice $\alpha = 1$ corresponds to a natural notion of light vector meson dominance. This choice sets to zero the coefficient of $v_\mu = i/(2F_\pi^2)(\phi\partial_\mu\phi - \partial_\mu\phi\phi) + \dots$ so that two pseudoscalars in a p-wave state can only be emitted through an intermediate light vector particle from a single heavy meson vertex. Whether, in fact, $\alpha \approx 1$ remains to be determined.

For our present application we also require the four fermion effective weak interaction:

$$\begin{aligned}\mathcal{L}_W &= \frac{G_F}{\sqrt{2}} J_\mu^{(+)} J_\mu^{(-)}, \\ J_\mu^{(-)} &= i\bar{\nu}_e \gamma_\mu (1 + \gamma_5) e + \dots, \\ J_\mu^{(+)} &= iV_{cs}^* \bar{s} \gamma_\mu (1 + \gamma_5) c + \dots,\end{aligned}\tag{2.3}$$

with usual conventions [3] and where V_{cs} is the Kobayashi-Maskawa matrix element. The chiral covariant realization of the left handed hadronic current, $J_\mu^{(+)}$ in terms of heavy and light meson fields is

$$J_\mu^{(+)} / V_{cs}^* = F_D [\partial_\mu D_b + i\alpha' \tilde{g} D_a \rho_{\mu ab} + i(1 - \alpha') D_a v_{\mu ab} + M D_{\mu b}^*] (\xi^\dagger)_{b3} + \dots, \tag{2.4}$$

where the SU(3) triplet fields (D_1, D_2, D_3) stand for $D^0, D^+ D_s^+$ and similarly for the heavy vectors. (Eq. (2.4) is the same as (4.6) of [3], but we have redefined $(\alpha + \alpha')$ by α' to avoid confusion). In (2.4) α' is a new dimensionless coupling constant (which scales however as M) characterizing the phenomenological hadron weak current. We can rewrite (2.4) in the heavy quark limit as

$$J_\mu^{(+)} / V_{cs}^* = \frac{-iF_D M}{2} \text{Tr} [\gamma_\mu (1 + \gamma_5) H_a] (\xi^\dagger)_{a3} + \frac{1}{2} F_D \alpha' \text{Tr} (\gamma_5 H_a) (\tilde{g} \rho_{\mu ab} - v_{\mu ab}) (\xi^\dagger)_{b3} + \dots \tag{2.5}$$

Now let us compute the hadronic matrix element for the process $D^0(p) \rightarrow K^-(p') + \pi^0(p'') + e^+(q_e) + \nu_e(q_\nu)$. We define the 4-momentum

$$q = q_e + q_\nu = p - p' - p'', \tag{2.6}$$

and employ the following form factor decomposition:

$$\begin{aligned}& \sqrt{8p_0 p'_0 p''_0} \langle K^-(p') \pi^0(p'') | J_\mu^{(+)} / V_{cs}^* | D^0(p) \rangle \\ &= -i[q_\mu r + (p' + p'')_\mu \omega_+ + (p' - p'')_\mu \omega_- + h \epsilon_{\mu\alpha\rho\nu} p_\rho p'_\nu p''_\alpha].\end{aligned}\tag{2.7}$$

The first term on the right hand side of (2.7) does not contribute to the net weak amplitude since q_μ dotted into the leptonic factor vanishes for zero lepton masses. Hence the form factor r is irrelevant. There are contributions to the form factors both with and without intermediate light vector meson K^* poles. The “non-resonant” (NR) diagrams without the intermediate K^* are discussed in the Appendix. Taking the leading order in M contribution to each form factor yields

$$\begin{aligned}(\omega_+)_{NR} &= \left(\frac{F_D}{2\sqrt{2}F_\pi^2} \right) \frac{dM}{\Delta - V \cdot p''}, \\(\omega_-)_{NR} &= \left(\frac{F_D}{2\sqrt{2}F_\pi^2} \right) \left(-\frac{dM}{\Delta - V \cdot p''} + \alpha' \right), \\h_{NR} &= \frac{F_D d^2}{\sqrt{2}F_\pi^2} \frac{1}{\Delta_s^* - V \cdot (p' + p'')} \frac{1}{\Delta - V \cdot p''},\end{aligned}\tag{2.8}$$

where $V_\mu = p_\mu/M$ is the 4-momentum of the initial D^0 , $\Delta = M(D^*) - M(D)$ and $\Delta_s^* = M(D_s^*) - M(D)$. ω_+ and ω_- both scale as $M^{1/2}$ while h (since it multiplies p_ρ) scales as $M^{-1/2}$. Eq. (2.8) differs from an earlier calculation [1] in the model with light *pseudoscalars* only by the α' term in $(\omega_-)_{NR}$. α' is due to the presence of light vectors, as may be seen from (2.4). Chiral covariance demands an additional pseudoscalar piece when we add the light vectors. The analogous “strong” parameter α does not appear in (2.8) but does show up in the form factor r_{NR} given in (A7).

The computation of the diagrams containing K^* poles can be simplified by making use of earlier results on the $D \rightarrow K^*$ weak current matrix element [2-6]. From section 5 of [3] we obtain the leading large M contribution in this case as

$$\begin{aligned}&\sqrt{4p_0k_0}\langle K^{*-}(k, \epsilon)|J_\mu^{(+)}V_{cs}^*|D^0(p)\rangle \\&= \sqrt{4p_0k_0}\langle \bar{K}^{*0}(k, \epsilon)|J_\mu^+V_{cs}^*|D^+(p)\rangle \\&= iF_D\bar{\epsilon}_\nu \left[\alpha'\tilde{g}\delta_{\mu\nu} + \frac{2cM}{m_v}\epsilon_{\sigma\nu\mu\beta} \frac{V_\beta k_\sigma}{\Delta_s^* - V \cdot k} - \frac{\alpha\tilde{g}V_\nu q_\mu}{\Delta_s - V \cdot k} \right],\end{aligned}\tag{2.9}$$

wherein $q_\mu = p_\mu - k_\mu$. Additional corrections due to higher derivative interactions [3,4], loops [4] and excited heavy states [2,6] have been discussed in the literature but (2.9)

seems sufficient for our present purpose. The pieces of the $D^0(p) \rightarrow K^-(p') + \pi^0(p'')$ transition matrix element in (2.7) involving K^* poles can be found from (2.9) simply by defining

$$k_\mu = p'_\mu + p''_\mu \quad (2.10)$$

and replacing $\bar{\epsilon}_\nu$ by the factor,

$$\mathcal{F}_\nu = \frac{\tilde{g}}{2\sqrt{2}} \left(\frac{m_v}{F_\pi \tilde{g}} \right)^2 \frac{[p'_\nu - p''_\nu - k_\nu(m_K^2 - m_\pi^2)/m_{K^*}^2]}{k^2 + m_{K^*}^2 - im_{K^*}^* \Gamma_{K^*}}, \quad (2.11)$$

wherein the combination $(\frac{m_v}{F_\pi \tilde{g}})^2$ is numerically close to 2.0. Eq. (2.11) is the product of the vector -2 pseudoscalar coupling constant and the K^* propagator. Since the K^* can go “on shell” we include the conventional width correction in the denominator to maintain unitarity. Decomposing the “resonant” K^* pole amplitudes (subscript R) into the form factors defined in (2.7) gives:

$$\begin{aligned} (\omega_+)_R &= -\frac{m_K^2 - m_\pi^2}{m_{K^*}^2} (\omega_-)_R, \\ (\omega_-)_R &= -\left(\frac{F_D}{2\sqrt{2}F_\pi^2} \right) \frac{\alpha' m_v^2}{k^2 + m_{K^*}^2 - im_{K^*}^* \Gamma_{K^*}}, \\ h_R &= \frac{2c\tilde{g}F_D}{\sqrt{2}m_v} \left(\frac{m_v}{F_\pi \tilde{g}} \right)^2 \frac{1}{\Delta_s^* - V \cdot (p' + p'')} \frac{1}{k^2 + m_{K^*}^2 - im_{K^*}^* \Gamma_{K^*}} \end{aligned} \quad (2.12)$$

The net results for the $D^0 \rightarrow K^-\pi^0$ current transition form factors in the limit of heavy charmed particles interacting with “soft” light pseudoscalars and vectors are given by the sums of the corresponding terms appearing in (2.8) and in (2.12). The precise range of validity of the concepts of a soft light pseudoscalar and (especially) a soft light vector are probably best left to a comparison with experiment. Certain results, which depend only on the existence of spontaneously broken chiral symmetry, are designated “current algebra theorems” and should hold for zero mass pseudoscalars as their 4-momenta go to zero. In particular we may note that $(\omega_+)_R$ vanishes in this limit and that $(\omega_-)_R$ (when we set $m_v^2 = m_{K^*}^2$ and $\Gamma_{K^*} = 0$, the latter corresponding to a pure effective Lagrangian computation which thereby satisfies chiral symmetry by construction) cancels off the α'

piece of $(\omega_-)_{NR}$. In this unphysical limit $\omega_+ = -\omega_-$ [9]. We should stress that the fact that α' cancels out in this limit does not mean that it is not an important parameter for describing the decay; actually, it turns out to be the most important parameter.

To proceed with a comparison of the resonant and non-resonant contributions we need at least a rough idea of the magnitudes of the “strong” parameters α, c and d as well as the “weak” parameter α' . Bounds on the value of d have been obtained in the literature [10] which agree with a simple estimate [3] based on pole dominance of the $D \rightarrow K$ transition form factor:

$$d \approx 0.53. \quad (2.13)$$

Information about c and α' may be obtained by comparing (2.9) with experimental information on the decays $D \rightarrow K^* e^+ \nu_e$. Eq. (2.9) is expected to be most reliable for “soft” K^* ’s, which implies that $(-q^2)$ should be as large as kinematically possible. Now the experimental data is analyzed in terms of form factors characterizing (2.9) which have the q^2 dependence, $M_s^{*2}/(M_s^{*2} + q^2)$. Evaluating (2.9) at the extreme value $-q^2 = (M - m_{K^*})^2$, *assuming* the above q^2 dependence to extrapolate to small $-q^2$ and comparing with the experimental values [11] yields the estimates

$$|\alpha'| \approx 1.73, \quad |c| \approx 1.6. \quad (2.14)$$

α is not determined from this analysis. However a study of the binding energy of heavy baryons as solitons of the meson Lagrangian involving (2.1) shows that (5.3) of Ref. [12] may be fit with the choice

$$d = 0.53, \quad c = 1.6, \quad \alpha = -2. \quad (2.15)$$

We stress that the numerical estimates (2.13)-(2.15) are preliminary in nature; however they should suffice to draw qualitative conclusions.

3. Comparison of Light Pseudoscalar and Light Vector Amplitudes

First we must discuss the kinematics associated with the hadronic matrix element (2.7). For the decay $D^0(p) \rightarrow K^-(p') + \pi^0(p'') + e^+(q_e) + \nu_e(q_\nu)$ there are five independent dynamical variables needed to specify the momentum configuration; a recent discussion is given in Ref. [1] (see also [13]) based on earlier treatments [14] of k_{e4} decays. It is somewhat simpler if we confine our attention just to the hadronic part (2.7). Then, for each value of the invariant lepton squared mass $-q^2$ we have in effect a three body final state. The classic Dalitz plot analysis shows that the kaon energy E' and the pion energy E'' (both in the D^0 rest frame) are sufficient. Altogether this gives E', E'' and q^2 as a possible complete set of variables to describe (2.7). Since k^2 plays an important role in the K^* pole amplitude we shall choose the alternative set

$$(k^2, q^2, E''). \quad (3.1)$$

A convenient formula relating the two sets is

$$E' + E'' = \frac{M^2 - k^2 + q^2}{2M} \equiv Q(k^2, q^2), \quad (3.2)$$

where M is the D^0 mass. With the neglect of the lepton masses, $-k^2$ satisfies

$$(m_\pi + m_k)^2 \leq -k^2 \leq M^2. \quad (3.3)$$

For a given k^2 , q^2 must satisfy

$$0 \leq -q^2 \leq (M - \sqrt{-k^2})^2. \quad (3.4)$$

The region defined by (3.3) and (3.4) is illustrated in Fig. 1. Now, for each value of q^2 there is a Dalitz plot boundary in the $k^2 - E''$ plane obtained by rewriting the condition $|\mathbf{p}' \cdot \mathbf{p}''| = |\mathbf{p}'||\mathbf{p}''|$ in terms of the set (3.1). The allowed regions, corresponding to

horizontal slices perpendicular to the page in Fig. 1, are illustrated in Fig. 2. A simple formula can be given for the “average” value of E'' at each (k^2, q^2) :

$$\begin{aligned}\overline{E''}(k^2, q^2) &= \frac{1}{2} [E''_{\max}(k^2, q^2) + E''_{\min}(k^2, q^2)] \\ &= \frac{1}{2} Q(k^2, q^2) \left[1 + \frac{m_K^2 - m_\pi^2}{k^2} \right].\end{aligned}\tag{3.5}$$

Notice that Fig. 1 does not correspond to a *fixed* E'' slice but rather represents the *projection* of the (k^2, q^2, E'') boundary surface into the (k^2, q^2) plane.

The expressions for the form factors in the heavy quark limit given by (2.8) + (2.12) are expected to be most reliable for large $(-q^2)$, corresponding to “soft” π^0 and K^- particles as well as, in the spirit of the present Lagrangian, soft K^* particles. Referring to Fig. 1, we see that the soft K^* condition suggests that we consider the effective Lagrangian expressions to hold in the region where $-q^2$ is greater than around 0.5 GeV^2 . This represents a fairly large portion of the entire phase space. Nevertheless it is not unreasonable from an experimental point of view inasmuch as the Particle Data Group tables state [15] that “it is generally agreed that the $\overline{K}\pi e^+\nu_e$ decays of the D^\dagger and D^0 are dominantly $\overline{K}^* e^+\nu_e$.” Experimentally, it is also known that the amplitudes are damped for decreasing $(-q^2)$; this can be seen from Fig. 1 to decrease the importance of the larger $-k^2$ region.

From a theoretical point of view it is very interesting to compare the pseudoscalar and vector contributions to the hadronic form factors. Nature tells us, of course, that the vector contributions dominate when one folds in the leptons and integrates their “squares” over all phase space. Nevertheless, it is important to get an idea of the “local” ratios of the two contributions in various kinematic regions. The phenomenological analysis of Ref. [16], for example, shows that it is the form factor proportional to $\bar{\epsilon}_\mu$ in (2.9) (i.e, the α' term) which mainly supplies the total width for $D^0 \rightarrow K^{*-} e^+ \bar{\nu}_e$. This form factor contributes to both $(\omega_+)_R$ and $(\omega_-)_R$ in (2.12) but $(\omega_-)_R$ is clearly the dominant one;

$(\omega_+)_R$ would vanish when the k and the π masses are equal or neglected. Hence we will focus our attention on the ω_- form factor. It is convenient to rewrite it as follows (in the D^0 rest frame):

$$\omega_- = \frac{F_D}{2\sqrt{2}F_\pi^2} \left[\alpha' \left(1 - \frac{m_{K^*}^2}{k^2 + m_{K^*}^2 - im_{K^*}\Gamma_{K^*}} \right) - \frac{dM}{\Delta + E''} \right]. \quad (3.6)$$

We noted earlier that the coefficient of the α' term (which corresponds to the extra piece introduced by adding light vectors to the effective Lagrangian) vanishes in the unphysical $k_\mu \rightarrow 0$ limit (with $\Gamma_{K^*} = 0$) in agreement with expectations. Let us then consider the ratio of the magnitude of the α' term to the magnitude of the last term:

$$\text{ratio} = \left| \frac{\alpha'}{d} \right| \frac{(\Delta + E'')}{M} \left[\frac{k^4 + m_{K^*}^2 \Gamma_{K^*}^2}{(k^2 + m_{K^*}^2)^2 + m_{K^*}^2 \Gamma_{K^*}^2} \right]^{1/2}. \quad (3.7)$$

Only α' and d are not very well known; we will use the estimates in (2.13) and (2.14) which are certainly qualitatively reasonable. There is a lower bound for the above,

$$\text{ratio} \geq \left| \frac{\alpha'}{d} \right| \frac{\Delta + m_\pi}{M} \frac{(m_K + m_\pi)^2}{m_{K^*}^2 - (m_K + m_\pi)^2} \approx 0.5, \quad (3.8)$$

where Γ_{K^*} was set to zero for simplicity since it has a negligible effect when $(-k^2)$ is as small as possible. This result indicates that there is no region in which the vector contribution is negligible compared to the pseudoscalar contribution. So if one were to make the usual derivative expansion of the chiral perturbation theory approach, there would be large corrections to the first order results due to the existence of the K^* . Of course, the pseudoscalar piece takes on its largest value near the cusp in Fig. 1.

Since (3.7) depends only on k^2 and E'' it is convenient to display curves of constant ratio in Fig. 2. We notice that there is only a very small region for which the ratio is less than unity. Because of the small width of the K^* (50 MeV) the ratio rises dramatically to around 15 at $-k^2 = m_{K^*}^2$. It is seen that the large ratio of amplitudes persists over a non-negligible region.

The ratio $\left| \frac{h_R}{h_{NR}} \right|$ of the magnitudes of the vector and pseudoscalar contributions to the weak vector current form factor is seen from (2.8) and (2.12) to be the same as the ratio (3.7) when we multiply the latter by the factor

$$2 \left| \frac{c}{\alpha' \tilde{g}} \right| \frac{m_{K^*} M}{(k^4 + m_{K^*}^2 \Gamma_{K^*}^2)^{1/2}} . \quad (3.9)$$

This is numerically around 1.0 in the phase space region of interest so the K^* contribution is also dominant for this form factor.

To sum up, in the large $(-q^2)$ region (optimistically as large as $-q^2 \geq 0.5 \text{ GeV}^2$) the K^* contribution overwhelms the pseudoscalar contribution nearly everywhere. Even for the very largest $(-q^2)$, where the soft pseudoscalar results are expected to be most significant, the K^* amplitudes are relatively sizeable. This situation would seem to suggest the desirability of a modified chiral perturbation theory program in which both pseudoscalars and vectors are retained in the effective Lagrangian from the very beginning. Some recent discussion of this point of view has been given in Ref. [17]. In the case where one is considering a non-strange transition matrix element (like $B \rightarrow \pi\pi$) we should replace in (3.8), $(m_K + m_\pi)$ by $2m_\pi$ and m_{K^*} by m_ρ . Then there would be a very small region in which the pseudoscalar piece might be considered dominant, but the overall picture would be qualitatively similar.

4. Remarks

1. Of course, it would be a better approximation to deal with the B rather than the D meson as an example of a heavy field. Similarly it would be a better approximation to restrict the light particles to the non-strange ones. Thus a similar analysis (with basically identical formulas) would be somewhat cleaner for $\overline{B} \rightarrow \pi\pi$ current transitions rather than $D \rightarrow \overline{K}\pi$ transitions. Nevertheless there is at present much more experimental data for the $D \rightarrow \overline{K}\pi$ case and it is quite interesting in its own right.

2. As noted after (2.9), the formula we are using for the resonant contribution neglects a number of effects which are subleading from the light meson point of view but may be necessary to take into account if the relatively small form factor which would be proportional to p_μ in (2.9) is actually non-zero. However we expect that our approximation is sufficient for the purpose of comparing the relative strengths of the vector and pseudoscalar contributions.

3. The ratio of the non-resonant part of $\Gamma(D \rightarrow \overline{K}\pi e^+\nu_e)$ to $\Gamma(D \rightarrow \overline{K}^* e^+\nu_e)$ is of evident experimental interest. The precise meaning of this quantity depends on the manner in which it is extracted from experiment. Apparently, there is no universal method. The most straightforward way is simply to define the resonant contribution as everything within a certain band of $-k^2$ surrounding $m_{K^*}^2$. This definition has, however, the misleading feature that it counts non-resonant background near the peak as resonant. It is particularly easy to apply this definition to the present case when one makes the reasonable approximation that the *entire* amplitude is dominated by the K^* pole diagrams. As recently illustrated for a different decay in Ref. [13] both the phase space and the squared amplitudes factorize in this approximation so that one obtains

$$\Gamma(D^0 \rightarrow K^-\pi^0 e^+\nu_e) \approx \frac{\Gamma(K^{*-} \rightarrow K^-\pi^0)}{\Gamma_{K^*}} \Gamma(D^0 \rightarrow K^{*-} e^+\nu_e) \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dx}{x^2 + 1}, \quad (4.1)$$

where x is defined by $k^2 + m_{K^*}^2 = x\Gamma_{K^*}m_{K^*}$. If the “resonant” region is taken to be that range of $-k^2$ for which $|\sqrt{-k^2} - m_{K^*}| < N\Gamma_{K^*}$ then integrating (4.1) yields the non-resonant/resonant ratio to be about

$$\frac{\pi}{2 \tan^{-1}(2N)} - 1. \quad (4.2)$$

This gives a 19% non resonant contribution for $N = 2$ and 12% for $N = 3$. The extent to which the data obeys (4.2) as a function of N might be considered a measure of how good is the K^* dominance. The fact that we do get a non-resonant contribution at all with pole dominance is, of course, an artifact of its definition. However, it also illustrates

the difficulty in giving a meaningful *experimental* definition of the non-resonant/resonant ratio. Since the “theoretical” resonant and non-resonant amplitudes have the same order of magnitude *outside* the resonance region, the above numbers may give a satisfactory rough order of magnitude estimate for any reasonable definition of the ratio.

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Appendix

For the process $D^0 \rightarrow K^- \pi^0 e^+ \nu_e$ we require the hadronic matrix element

$$\sqrt{8p_0 p'_0 p''_0} \langle K^-(p') \pi^0(p'') | J_\mu^+ / V_{cs}^* | D_0(p) \rangle, \quad (A1)$$

where the states are normalized in a unit volume. We shall proceed by first using the ordinary, rather than “heavy”, meson fields to compute (A1) (see section V of Ref.[3]) and then take the heavy quark limit. The contribution to (A1) from the contact (non-pole) diagram is:

$$\frac{-iF_D}{2\sqrt{2}F_\pi^2} [p_\mu + (\alpha' - 1)(p'_\mu - p''_\mu)]. \quad (A2)$$

The D_s^+ pole diagram, wherein $D^0(p) \rightarrow K^-(p') + \pi^0(p'') + D_s^+(q)$ followed by $D_s^+(q) \rightarrow e^+ \nu_e$, contributes the term

$$\frac{iF_D(1 - \alpha)}{2\sqrt{2}F_\pi^2} \frac{(p' - p'') \cdot (p + q)}{M_s^2 + q^2} q_\mu, \quad (A3)$$

in which M_s is the D_s^+ mass and $q_\mu \equiv p_\mu - (p' + p'')_\mu$. The D^{*0} pole diagram, wherein $D^0 \rightarrow \pi^0 + D^{*0}$ at the strong vertex followed by $D^{*0} \rightarrow K^- + \mu^+ \nu_\mu$ at the weak vertex, contributes:

$$\frac{-i\sqrt{2}M^2 F_D d}{F_\pi^2} \frac{[p''_\mu + (p - p'')_\mu (p - p'') \cdot p'' / M^{*2}]}{(p - p'')^2 + M^{*2}}, \quad (A4)$$

in which M^* is the D^{*0} mass. There are also two double-pole diagrams. The first features $D^0 \rightarrow \pi^0 + D^{*0}$ at a strong vertex followed by $D^{*0} \rightarrow K^- + D_s^{*+}$ at another strong vertex which, in turn, is followed by $D_s^{*+} \rightarrow e^+ \nu_e$ at the weak vertex. The contribution to (A1) is

$$\frac{-i2\sqrt{2}d^2 M^2 F_D}{F_\pi^2} \frac{\epsilon_{\mu\alpha\rho\nu} p_\rho p'_\nu p''_\alpha}{[M_s^{*2} + q^2][M^{*2} + (p - p'')^2]}, \quad (A5)$$

in which M_s^* is the D_s^{*+} mass. Finally, the diagram with $D^0 \rightarrow \pi^0 + D^{*0}$, $D^{*0} \rightarrow K^- + D_s^+$, $D_s^+ \rightarrow e^+ \nu_e$ gives

$$\frac{-i2\sqrt{2}F_D M^2 d^2}{F_\pi^2} \frac{q_\mu [p' \cdot p'' + p' \cdot (p - p'') p'' \cdot (p - p'') / M^{*2}]}{[q^2 + M_s^2][(p - p'')^2 + M^{*2}]}. \quad (A6)$$

In order to obtain a model independent result corresponding to $M \rightarrow \infty$ we should delete terms which fall off faster than $M^{1/2}$, the scale behavior of (A1). For example, using $F_D \sim M^{-1/2}$, and $\alpha' \sim M$, (A2) \rightarrow

$$\frac{-iF_D}{2\sqrt{2}F_\pi^2}[p_\mu + \alpha'(p' - p'')_\mu]. \quad (A2')$$

In taking the limit of (A3) we set $p_\mu = MV_\mu$ and throw away terms of quadratic order in p' and p'' . The resonance denominator of (A3) becomes $2M[\Delta_s - V \cdot (p' + p'')]$ with $\Delta_s = M_s - M$ and (A3) \rightarrow

$$\frac{-iF_D(1 - \alpha)}{2\sqrt{2}F_\pi^2} \frac{V \cdot (p' - p'')}{\Delta_s - V \cdot (p' + p'')} q_\mu. \quad (A3')$$

The other diagrams are treated similarly. With the form factor decomposition defined in (2.7) we obtain the results listed in (2.8) for the three experimentally significant form factors. For the sake of completeness we also give here:

$$r_{NR} = \frac{F_D}{\sqrt{2}F_\pi^2} \left[\frac{1}{2} + \frac{(1 - \alpha)}{2F_\pi^2} \frac{V \cdot (p' - p'')}{\Delta_s - V \cdot (p' + p'')} + \frac{dV \cdot p''}{\Delta - V \cdot p''} + \frac{d^2(p' \cdot p'' + V \cdot p'V \cdot p'')}{[\Delta_s - V \cdot (p' + p'')](\Delta - V \cdot p'')} \right]. \quad (A7)$$

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Figure Captions

Fig. 1 Projection of the three dimensional (k^2, q^2, E'') phase space boundary into the (k^2, q^2) plane.

Fig. 2 Phase space boundaries in the $k^2 - E''$ plane at various values of q^2 . In increasing order of size the closed curves correspond respectively to $-q^2 = 0.925, 0.725, 0.525, 0.325$ and 0.025 GeV^2 . Also shown are points on the contour lines on which the ratio in (3.7) takes on fixed values; the circles, crosses and squares correspond respectively to the ratio equal to 1, 3 and 9.